Dynamitic Analysis of Skid Steering for Articulated Motor-driven Vehicle

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Abstract
The hydraulic steering mechanism can lead the vehicle’s hydraulic system more complex and the tire wear heavier in the process of steering, which is detrimental to the steering control of the articulated motor-driven vehicle. Thus the skid steering mode is implemented in the articulated motor-driven vehicle in this paper. This mode can simplify the vehicle without steering cylinder and it is more flexible by controlling the velocity of each wheel when steering. According to the theory of traditional wheeled vehicle’s skid steering and based on the analysis of skid steering of articulated vehicle, a dynamic model is established to describe the mathematic relationships among the steering radius, body structure, velocity of front and rear wheels, longitudinal forces and lateral forces between each tire and the ground. With these analyses, the kinematic and dynamic simulations based on the model are applied according to the parameters of an underground mine articulated vehicle. And the results of the simulation can verify the model, which can provide references for designing and studying skid steering of articulated motor-driven vehicle.

Keywords: Articulated Motor-driven vehicle, Skid Steering, Model, Simulation

1 Introduction
The common steering mode of articulated engineering vehicle is hydraulic steering. It can meet the steering needs in different conditions, but there still exist several problems in body structure and response speed of steering. Firstly, the tyre is heavy wear because of the slipping of wheels in the process of steering [1]. Secondly, the steering of the articulated vehicle relies mainly on the hydraulic cylinder and articulated joint body, which the force act on its can not be ignored [2]. Thirdly, the biggest steering angle of the articulated vehicle is limited by hydraulic cylinder, which is generally about 45 degrees [3]. Fourthly, there still exist problems in steering response due to the delay action of the hydraulic oil.

Compared with the hydraulic steering mode, skid steering mode does not rely on the steering mechanism but use the different speeds of wheels at both sides to alter the driving direction [4]. The vehicle with skid steering can implement in-situ steering and zero-radius steering, which can greatly improve wheeled vehicle steering properties and improve its maneuverability. The tyre of articulated vehicle with this kind of steering mode is generally in a state of lateral deviation when steering, which has less tyre wear [5]. Currently, skid steering mode is mainly used in crawler vehicles, skid steer loaders and some multi-axis vehicles. In literature [4-9], the skid steering mode is designed by analyzing the steering principle of different vehicles to study the force characteristics of each wheel in the skid steering process. However, none of these
researches analyze the skid steering of articulated vehicle or connect the characteristic that the motor-driven wheel could be controlled independently with skid steering. Therefore, it can not provide references for the studying and designing of skid steering of articulated vehicle.

In this paper, skid steering mode is applied in articulated vehicle of which the wheels can be controlled independently to simplify the body structure. By analyzing the skid steering process of the front and rear part and taking the minimum force of the articulated joint body as a target, a kinematic model of articulated vehicle is established to study the motion characteristics of wheels at a certain vehicle velocity. In addition, the front part is taken as a research object to discuss the dynamic characteristics of skid steering and study the inertia force, steering resisting moment and the wheel force produced by equivalent axle. Based on the single-axle model, the relations among the driving force, the velocity of vehicle and the rate of rear wheels’ velocity change are derived. At last, the dynamic and kinematic simulations with the specification of a 10t underground mine articulated vehicle with motor-driven wheels are implemented to verify the model, which the results can provide references for the designing and controlling of the skid steering articulated vehicle.

2 Kinematic Analysis of Skid Steering for Articulated Vehicle

There are four steering steps when the articulated vehicle with skid steering mode turns left and gets through the curve with radius $R$ at a certain vehicle velocity $v$ (In the following steering analyses, the default steering direction is left if not mentioned specially). Firstly, the rate of velocity change of rear wheels is matched with the reference of vehicle velocity $v$ and palstance of steering wheel. Secondly, the velocity of front wheels is controlled with the target of taking the minimum force of the articulated joint body. Thirdly, the steering angle $\theta$ is established which suits the turning radius $R$ of articulated vehicle. At last, the articulated vehicle gets through the curve. The steering process is as following in Fig.1.

With the reference of Fig.1, a kinematic model of articulated vehicle at time $t$ is established in Fig.2 to analyze the process of wheels’ change and steering angles’ make. When the articulated vehicle with skid steering mode steer on the purpose of taking the minimum force of the articulated joint body, the velocity of rear wheels is active controlled and the front wheel velocity is controlled based on the particular motion trajectory at joint part, which could make the articulated joint body of the front part turning with that of the rear part. In the process of steering, the line of instantaneous steering center of front and rear part which is represented by $O_A$ and $O_B$ goes through the point $O$. And the velocity of front part $V_{01}$ is the same as that of the rear part represented by $V_{02}$ whose direction is vertical to the line of $O_A$ and $O_B$ at point $O$.

In Fig.2, a local coordinate system denoted $x_1-O-y_1$ is assigned to the front part at the center of articulated joint body $O$, of which the positive direction of $x_1$ is the right direction of the articulated joint body vertical to the front part, and the positive direction of $y_1$ is the forth direction of the articulated joint body parallel to the front part; $A, B, C, D$ respectively represented left front wheel, right front wheel, rear wheel of right side and left side; $E_1$ and $E_2$ are the center of front and rear axle (Articulated vehicle with motor-driven wheels has a virtual axle which is the equivalent of the vehicle frame.); $V_i$ describes the velocity of $i$th wheel; $\alpha$ and $\beta$ represent the steering angles of front and rear part when the $\theta$ represent the steering angle of articulated vehicle; $\gamma$ is the angle of instantaneous steering center $O_B$ and rear part of the articulated vehicle; $\omega_1$ and $\omega_2$ are the palstance of the front and rear part; $L_1$ and $L_2$ respectively represent the wheel base which are the distances between the front axle center, the rear axle center and the articulated joint body; $W$ is the wheel track; $R_1$ and $R_2$ are the instantaneous steering radius of front
and rear part which are simplified steering radius \( R_1 \) and \( R_2 \) in the following analyses.

![Fig.2 Steering kinematic model of articulated vehicle](image)

### 2.1 Analysis of Skid Steering for Rear Part

According to the skid steering analyses of articulated vehicle above, the velocity of rear wheels at time \( t \) can be expressed by Eq. (1) when the difference of rear wheels velocity change in liner way.

\[
\begin{align*}
V_C &= v \\
V_D &= v - \lambda t
\end{align*}
\]  

(1)

where, \( \lambda \) represents the rate of velocity change of rear wheels which determined by the velocity of articulated vehicle and the palstance of steering wheel.

The palstance of rear part \( \omega_2 \) can be obtained by Eq. (2).

\[
\omega_2 = \frac{d\beta}{dt} = \frac{V_C - V_D}{W} = \frac{\lambda t}{W}
\]  

(2)

With the combination of structure parameters of articulated vehicle and Eq. (2), the velocity of rear wheels can also be expressed by Eq. (3).

\[
\begin{align*}
V_C &= \omega_2 \cdot \left( R_2 + \frac{W}{2} \right) \\
V_D &= \omega_2 \cdot \left( R_2 - \frac{W}{2} \right)
\end{align*}
\]  

(3)

The steering radius \( R_2 \) could be obtained from Eq. (4) according to the Eqs. (1) - (3).

\[
R_2 = \frac{V_C + V_D}{2(V_C - V_D)} = \frac{(2v - \lambda t)W}{2\lambda t}
\]  

(4)

So the velocity of rear part at joint body \( O \) could be obtained from Eq. (5).

\[
V_{O2} = \sqrt{\frac{\lambda^2 t^2 L_2^2}{W^2} + \frac{(2v - \lambda t)^2}{4}}
\]  

(5)

The velocity of rear part along \( x_1 \) and \( y_1 \) direction in the local coordinate system \( x_1-O_{1-y_1} \) could be expressed by Eq. (6).

\[
\begin{align*}
V_{O2x1} &= V_{O2} \cdot \cos(\zeta) \\
V_{O2y1} &= V_{O2} \cdot \sin(\zeta)
\end{align*}
\]  

(6)

where, \( V_{O2x1} \) and \( V_{O2y1} \) are the velocity of front part at joint body along \( x_1 \) and \( y_1 \) direction in the local coordinate system \( x_1-O_{1-y_1} \), \( \zeta \) is the angle of instantaneous steering center \( O_2 \) and front part which can be expressed by Eq. (7).

\[
\zeta = \frac{\pi}{2} - \theta - \gamma
\]  

(7)

In the Eq. (7), \( \theta \) is the steering angle of articulated vehicle which includes \( \alpha \) and \( \beta \); \( \gamma \) is the angle of instantaneous steering center \( O_2 \) and rear part whose value is related to the steering radius \( R_2 \). These two angles can be expressed by Eq. (8).

\[
\begin{align*}
\theta &= \alpha + \beta \\
\gamma &= \arctan \left( \frac{L_2}{R_2} \right)
\end{align*}
\]  

(8)

### 2.2 Analysis of Skid Steering for Front Part

For the front part of the articulated vehicle, the palstance \( \omega_1 \) can be expressed by velocity of front wheels which is shown in Eq. (9).

\[
\omega_1 = \frac{d\alpha}{dt} = \frac{V_b - V_d}{W}
\]  

(9)

The velocity of joint body \( O \) at front part is shown in Eq. (10).

\[
V_{O1} = \omega_1 \cdot \frac{L_i}{\cos(\zeta)}
\]  

(10)

With the combination of structure parameters of articulated vehicle and Eqs.(4) and (9) and (10), the velocity of front part along \( x_1 \) and \( y_1 \) direction in the local coordinate system \( x_1-O_{1-y_1} \) could be expressed by Eq. (11).

\[
\begin{align*}
V_{O1x1} &= \frac{V_b - V_d}{W} \cdot L_i \\
V_{O1y1} &= \frac{V_b - V_d}{2}
\end{align*}
\]  

(11)

where, \( V_{O1x1} \) and \( V_{O1y1} \) are the velocity of rear part at joint part along \( x_1 \) and \( y_1 \) direction in the local coordinate system \( x_1-O_{1-y_1} \).

When the velocity of front part controlled by the motion trajectory of articulated joint body, the velocity \( V_{O1} \) and \( V_{O2} \) should have the same value along \( x_1 \) and \( y_1 \) direction in the local coordinate system \( x_1-O_{1-y_1} \). Thus the velocity of the front
wheels could be expressed by Eq. (12) with the combination of Eqs. (6) and (11).

\[
\begin{align*}
V_A &= V_{02} \sin(\zeta) - \frac{W}{2L_1} V_{02} \cos(\zeta) \\
V_B &= V_{02} \sin(\zeta) + \frac{W}{2L_1} V_{02} \cos(\zeta)
\end{align*}
\]  
(12)

2.3 Analysis of Skid Steering Angle θ for Articulated Vehicle

Substituting Eq. (12) into Eq. (9) and combining the Eqs. (2) and (8), the steering angle θ of articulated vehicle could be obtained from Eq. (13).

\[
\theta = \int \frac{V_{02} \cdot \cos(\zeta)}{L_1} dt + \int \frac{\dot{\lambda} t}{W} dt
\]  
(13)

\[
\theta + \gamma = \int \frac{V_{02} \cdot \sin(\theta + \gamma)}{L_1} dt + \int \frac{\dot{\lambda} t}{W} dt + \arctan \left( \frac{2L_2 \lambda t}{(2v - \lambda t) \cdot W} \right)
\]  
(14)

\[
\zeta' = \frac{V_{02} \cdot \sin \zeta}{L_1} + \frac{\lambda t}{W} + \frac{2L_2 \lambda W (2v - \lambda t) + 2L_2 \lambda^2 W t}{(2v - \lambda t)^2 \cdot W^2 + 4L_2 \lambda^2 t^2}
\]  
(15)

With the analysis of Eq. (13), the steering angle θ of vehicle is related to the angle ζ which include angle θ in return according to the Eq. (7). Substituting Eq. (7) into Eq. (13) and combining the Eqs. (4) and (8), the angles θ and γ could be expressed by Eq. (14).

By differentiating the Eq. (14), the steering angles θ and γ could be simplified into Eq. (15) when making θ + γ = ζ.

According to the Eq. (15), the value of steering angle θ is related to the time t, the velocity of articulated vehicle v, the rate of velocity change λ, and the body structure of articulated vehicle. There is no analytical solution about articulated vehicle angle θ but we can get the arithmetic solution with the combination of Eqs. (5) and (15).

3 Dynamic Analysis of Skid Steering for Articulated Vehicle Based on the Single-axle Model

Based on the kinematic analyses of vehicle in section 2, the velocity of each wheel should meet the special relations to minimize the force of the articulated joint body in the steering process. In this condition, the front and rear part will be mutual independence and its trajectory is a circle which the circle center is at the respective instantaneous center O_A and O_B. The motion trajectories of articulated joint body, front and rear parts of vehicle are shown in Fig.3.

![Fig.3 The motion trajectory of joint body, front and rear part of vehicle](image)

With the analyses of Figs.3 and Eqs.(1) and (12), the difference of two wheels in front and rear part will increase with the steering of articulated vehicle, which make the instantaneous steering center O_A and O_B move toward the center of axle. The velocity of front and rear part at articulated joint body O is similar whose motion direction is the tangential direction of front and rear part movement. Therefore, the dynamic analysis of front and rear part should be considered alone when the loads of the articulated vehicle are bore by wheels and ignore the force of articulated joint body.

Because the movements of the front and rear part are mutually independent, force analysis should be different. When taking the front part as an example to analyse the articulated vehicle steering, the steering process and the slip or deflection angle of front wheels are shown in Fig.4. Based on the analyses of Fig.4, the steering process of two wheels in front part could be divided into two steps, establishing deflection angle and steady steering. The establishing deflection angle represents the lateral deviation degree which generated by the different velocity of two wheels under the condition of steering radius R_1 of front part; the steady steering represents the process of establishing the steering angle α of front part to adapt the steering radius R of articulated vehicle. Due to the tyre deflection angle is small and the moving distance is short before the steady steering
of front part, the steering angle in the process of establishing deflection could be neglected and at the moment of steady steering stage begin, the tread is always parallel to the movement direction of front part.

According to the analysis of steering process above, the force of each wheels in front part could be shown in Fig.5

In Fig.5, \( x-O_m-y \) is a model coordinate system, of which the positive direction of \( x \) is the right level direction and the positive direction of \( y \) is the forth level direction; \( x_2-O_2-y_2 \) is a local coordinate system assigned to the front part at instantaneous steering center \( O_2 \), of which the positive direction of \( x_2 \) is the right direction and vertical to the front part when the positive direction of \( y_2 \) is the forth direction and parallel to the front part; \( F_i \) is the driving force of each wheels; \( F_\beta \) is rolling resistance force; \( T_\beta \) is steering resistance moment; \( F_g \) is the centrifugal force; \( F_m \) is the lateral force produced by front axle.

With the analysis of Fig.5, the driving and resistance moment of front wheels should be balanced at instantaneous steering center \( O_2 \), which is shown in Eq. (16).

\[
\left( F_i - F_{\beta} \right) \left( R - \frac{W}{2} \right) \cos \xi_2 + \left( F_\beta - F_m \right) \left( R + \frac{W}{2} \right) \cos \xi_2 = T_\beta + T_{\beta m}
\]

(16)

3.1 Force Analysis of Front Wheels

3.1.1 Load Distribution of Front Part

The change of the tire load caused by the centrifugal force can not be neglected when the articulated vehicle steer at a certain velocity \( v \). In order to get the load distribution of the tire caused by its inertial force, the sketch of load distribution is established in Fig.6 without the condition of the mass center deflection.

In Fig.6, \( x-O_m-z \) is the coordinate system of front part, of which the positive direction of \( x \) is the right direction, and the positive direction of \( z \) is the upward direction vertical to the ground. \( F_{ZA} \) and \( F_{ZB} \) are the vertical force of right and left wheel of front part. \( F_{ZA} \) and \( F_{ZB} \) are the lateral resistance force generated by centrifugal force. \( O_s \) is the mass center of front part; \( h \) is distance between the mass center and ground; \( G \) is the gravity of the front part. In terms of the load distribution shown in the Fig.6 and the force analysis of the front part, the vertical force \( F_{ZA} \) and \( F_{ZB} \) of the tire can be expressed by the Eq. (17) based on the balance of the force and moment.

\[
\begin{align*}
F_{ZA} &= \frac{G}{2} \left( \frac{F_g}{2} - \frac{mg}{W} \right) = \frac{mv^2}{2RW} \\
F_{ZB} &= \frac{G}{2} \left( \frac{F_g}{2} + \frac{mg}{W} \right) = \frac{mv^2}{2RW}
\end{align*}
\]

(17)
3.1.2 Deflection angle $\xi_2$ and steering radius $R_1$ of front wheels

The angle between the tyre carcass and the tread is $\xi_2$ which is also the angle of tyre deflection. It is assumed that $1/n$ turns of the wheel will make the wheel steer $\xi_2$ degree. According to the recovery process of tyre and the steering characteristics, the relationship between the slip angle $\xi_2$ and the steering radius $R_1$ of front wheels is shown in Eq.(18).

$$R_1 \cdot \xi_2 = \frac{2\pi \cdot r_g}{n}$$ (18)

where, $r_g$ is the rolling radius of wheel, $n$ is the recovery coefficient of tyre.

3.1.3 Steering Resistance Moment $T_f$ and Rolling Resistance $F_f$  

With the analysis of Fig.4, the relations among slip angle $\xi_A$, deflection angle $\xi_2$ and torsion angle $\xi_t$ could be expressed in Eq. (19).

$$\xi_{1A} + \xi_{2B} = \xi_{2A} + \xi_{2B} + \xi_{4A} + \xi_{4B}$$ (19)

In the steering process, the slip angles $\xi_{4A}$ and $\xi_{4B}$ generated by centrifugal force are same in value, which is similar to the angles $\xi_{2A}$ and $\xi_{2B}$. So the steering resistance moment of tyre produced by torsional deformation could be obtained from Eq. (10) according to the literal [10] and Eqs. (18) and (19).

$$T_{fa} + T_{fb} = k \left(\frac{mv^2}{k \cdot R} + \frac{4\pi r_g}{n \cdot R}\right)$$ (20)

where, torsional stiffness of tyres $k' = \pi k r^{*'}$, $r'$ is the tyre radius of ground contact and $k$ is cornering stiffness of tyres.

When the articulated vehicle is moving on dry and even road which the rolling resistance coefficient is $f$, the longitudinal rolling resistance force $F_f$ is proportional to the vertical force of tyre $F_Z$ and the relation can be obtained from Eq. (21).

$$F_f = f \cdot F_Z$$ (21)

3.1.4 Lateral Force $F_L$ Produced by Front Axle

With the dynamic analyses above, the tread is parallel to the x-axis of the model coordinate system x-O-y at the moment of steady steering stage begins, therefore, the lateral deformation of the tyre carcass at the two sides are the same in value but opposite in direction, which can be obtained from Eq. (22).

$$\Delta_1 = -\Delta_2 = \frac{W}{2} \left(1 - \cos \xi_2\right)$$ (22)

Based on the analysis of Eq. (22), the axle lateral forces that the wheels bore are the same in value but opposite in direction which is the same with the analysis of lateral deformation of tyre carcass. According to literature [11] and lateral stiffness of tyre $\rho_y$, the lateral force $F_r$ could be derived in Eq. (9).

$$F_r = \rho_y \cdot \Delta = \rho_y \cdot \frac{W}{2} \left(1 - \cos \xi_2\right)$$ (23)

3.1.5 Driving Force $F$ of Front Wheels

According to the force diagram in Fig.5 and Eqs.(16) and (21), the driving force in front wheels could be shown in Eq. (24).

$$\begin{align*}
F_A &= f \cdot F_{ZA} = \frac{T_{fa} + T_{fb}}{W \cdot \cos(\xi_2)} \\
F_B &= f \cdot F_{ZB} = \frac{T_{fa} + T_{fb}}{W \cdot \cos(\xi_2)}
\end{align*}$$ (24)

3.2 Critical Steering Radius $R_{min}$

As can be seen in Fig.4, the steering of tread is lag behind the tyre carcass in the steering process of front wheels. Thus for the tyres, there are torsional and lateral deformation, which produce steering driving force to make the tyre steering. When the steering radius $R_1$ of front part is large, the deformation of tyre produces deformation angle $\xi_2$ including slip angle and torsional angle, which could meet the requirement of large steering radius. The steering of front wheels is completed in the state of tyre deformation. And when the steering radius $R_1$ of front part decreases, the maximum tyre deformation could not meet the requirement of the steering radius, the tyre is forced to steer by the torsional moment. In this case, it will happen slip steering. Therefore, there is a critical steering radius $R_{min}$ related to the minimum deformation between tread and tyre carcass in the steering process of front wheels. And this critical radius $R_{min}$ divides the radius $R_1$ into two parts, the wheel steers with the tyre deformation when the radius $R_1$ belong to $0-R_{min}$ and it will be slip steering when the radius $R \geq R_{min}$.

The minimum deformation of front wheels is connected with the friction resistance moment $T_{fmax}$ which is produced by the interaction of tyres and ground. According to the force analysis of tracked vehicle, the value of resistance moment $T_{fmax}$ could be obtained according to the Talblick Formula introduced in literature [12].
The contact surface between the tyres and ground can be simplified as a circle with the radius $r'$. If the wheel steers around the contact center, the relation between the steering resistance moment and the vertical force could be obtained from the Eq. (25).

$$T_{\text{max}} = F_z \cdot f \cdot K \quad (25)$$

where, $K$ is the equivalent radius of the contact circle which can be expressed by Eq. (26).

$$K^2 = \frac{I_p}{A} = \frac{r'^2}{2} \quad (26)$$

where, $I_p$ is the polar moment inertia when $A$ is the area of contact surface.

### 3.2.1 Critical Steering Radius $R_{A\text{min}}$ of Left Wheel $A$

With the analyses of section 3.1, the torsion angle and slip angle between inside wheel $A$ and outside wheel $B$ could be shown in Eq. (27).

$$\sqrt{2} f \left( mg W R_{A\text{min}} - 2m v^2 \right) = \frac{2 \pi r_g}{n R_{A\text{min}}} + \frac{mv^2}{2k R_{A\text{min}}} + \frac{\rho_y W \left( 1 - \cos \left( 2\pi r_g / n R_{A\text{min}} \right) \right)}{k} \quad (28)$$

$$\sqrt{2} f \left( mg W R_{B\text{min}} + 2m v^2 \right) = \frac{2 \pi r_g}{n R_{B\text{min}}} + \frac{mv^2}{2k R_{B\text{min}}} + \frac{\rho_y W \left( 1 - \cos \left( 2\pi r_g / n R_{B\text{min}} \right) \right)}{k} \quad (29)$$

### 4 Kinematic and Dynamic Simulation for Underground Mine Articulated Vehicle

According to the analyses above, kinematic and dynamic simulations of skid steering process for a 10t underground mine articulated vehicle to study the steering process and wheel forces. The parameters of this articulated vehicle are shown in Table 1.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Parameter</th>
<th>Value</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of front part</td>
<td>$m_1$(kg)</td>
<td>6000</td>
<td>Working on tyre</td>
</tr>
<tr>
<td>Mass of rear part</td>
<td>$m_2$(kg)</td>
<td>14000</td>
<td></td>
</tr>
<tr>
<td>Front wheelbase</td>
<td>(L_1)(mm)</td>
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<td>-</td>
</tr>
<tr>
<td>Rear wheelbase</td>
<td>(L_2)(mm)</td>
<td>2600</td>
<td>-</td>
</tr>
<tr>
<td>Tread</td>
<td>(W)(mm)</td>
<td>1450</td>
<td>-</td>
</tr>
<tr>
<td>Front part mass center</td>
<td>(h_1)(mm)</td>
<td>850</td>
<td>-</td>
</tr>
<tr>
<td>Rear part mass center</td>
<td>(h_2)(mm)</td>
<td>1200</td>
<td>-</td>
</tr>
<tr>
<td>Tyre rolling radius</td>
<td>(r_d)(mm)</td>
<td>810</td>
<td>-</td>
</tr>
<tr>
<td>Tyre ground radius</td>
<td>(r')(mm)</td>
<td>200</td>
<td>-</td>
</tr>
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<td>-</td>
</tr>
<tr>
<td>Tyre cornering stiffness</td>
<td>(k)/(N/mm)</td>
<td>-40000</td>
<td>-</td>
</tr>
<tr>
<td>Tyre deformation recovery coefficient</td>
<td>(n)</td>
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<tr>
<td>Adhesion coefficient</td>
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<td>Even road</td>
</tr>
<tr>
<td>Rolling resistance coefficient</td>
<td>(f)</td>
<td>0.02</td>
<td>-</td>
</tr>
</tbody>
</table>
4.1 Kinematic Simulation

When the articulated vehicle steers with the skid steering mode at the velocity \( v = 5 \text{ km/h} \), the rate of velocity change \( \lambda = 0.01 \) by controlling the tyre speeds of rear part. To assure the force of articulated joint body to be the minimum, the wheels speed of the front part should meet Eq. (12) which is shown in Fig.7.

![Fig.7 Velocity of front wheels](image)

When the wheels velocity of both front and rear part meets the above relationships, the change of steering angles of the vehicle, front and rear part with time \( t \) is shown in Fig.8.

![Fig.8 Steering angle of the articulated vehicle, the front and rear part](image)

Conclusions could be obtained from Fig.7 and Fig.8.

(1) In steering process, when the difference of rear wheels velocity is linear with the change rate \( \lambda = 0.01 \), the velocity of front wheels nonlinearly changes to assure the force of articulated joint body to be minimum. The left wheel velocity \( V_A \) will directly decline, and become negative when the vehicle steering angle is about 62 degrees. The right wheel velocity \( V_B \) is larger than the left and firstly increases then decreases in the steering process of articulated vehicle.

(2) The steering angle \( \theta \) of the articulated vehicle includes the front and rear steering angles, \( \alpha \) and \( \beta \). When the wheel velocity difference of the rear part changes linearly with the rate of velocity change \( \lambda = 0.01 \), the rear steering angle \( \beta \) increases with time \( t \) in a quadratic relation and the front steering angle \( \alpha \) increases nonlinearly. The steering angle of the front part is larger than that of the rear part.

(3) The steering speed of articulated vehicle with skid steering mode is faster than hydraulic steering mode when the articulated vehicle steers in the same condition. The articulated vehicle will steer generally about 50 degrees at 4.5s with the conditions of vehicle \( v = 5 \text{ km/h} \) and the rate of velocity change \( \lambda = 0.01 \).

According to the skid steering analyses in section 2, the changes of steering angle \( \theta \) depend on the vehicle velocity \( v \) and the rate of velocity change \( \lambda \) of the rear part. There are two types of simulations between the time \( t \) and steering angle \( \theta \) in the following analyses. One is about the same vehicle velocity and different rate of velocity change. The other is about the different vehicle velocity and same rate of velocity change. The simulation results are shown in Fig.9 and Fig.10.

![Fig.9 Steering angle of articulated vehicle with different rate of velocity change](image)
With the analyses of these two simulations, the results obtained from Fig.9 and Fig.10 are as following.

(1) When the articulated vehicle with skid steering mode steer at a certain velocity, different velocity change rate has different steering angle changing process. When the articulated vehicle steers to a same angle, the required time will decline with the rate of velocity change increasing.

(2) When the articulated vehicle with skid steering has the same rate of velocity change and different vehicle velocity, the time for the same steering angle of articulated vehicle is less if the velocity is larger.

Through the above simulation results, the value of vehicle steering angle depends on the vehicle velocity $v$ and velocity change rate $\lambda$ of the rear part in the process of steering. In actual skid steering operation, the articulated vehicle can steer smoothly by matching the rate of velocity change $\lambda$ according to the velocity of articulated vehicle and controlling its change with the palstance of steering wheel.

**4.2 Dynamic Simulation**

The front and rear part are independent according to the dynamic analyses in section 3. And its stress characteristics are the same without considering the impact of the articulated vehicle stress in the process of steering. When the dynamic simulations are based on the front part, the driving and rolling resistance forces are studied with different steering radius $R_1$ of the front part and articulated vehicle steering angle $\theta$. The simulation results are shown in Fig.11 and Fig.12.

Referring to the force simulations of Fig.11 and Fig.12, several results are made as following.

(1) The analysis process of steering force characteristic can be divided into several parts when the articulated vehicle with skid steering mode steers in a certain velocity. If the front steering radius is infinite and the vehicle drives straight, the driving forces of front part in two sides are equal in value which mainly used to overcoming the rolling resistance produced by the ground. And the vehicle begins steering when the front steering radius declines. There exist driving force differences at the front wheels due to the steering resistance moment and rolling resistance forces. The smaller of the steering radius are, the larger of the driving force difference will be. When the steering radius decline to the critical value $R_{min}$ and the vehicle steering angle is larger than $\theta_{min}$, the wheels begin slipping. The driving force moment produced by the axle will be larger than the maximum rolling resistance moment $T_{fmax}$.
produced by the ground. Thereafter, the driving force will not change with steering radius, but the rolling resistance will still influence the driving forces of front wheels with the steering radius declining.

(2) Slip steering begins earlier at the front inside wheel than the outside wheel. The minimum steering radius of outside wheels without slippin is 15.63m, which the corresponding minimum vehicle steering angle is 4.5 degrees when the minimum radius of inside wheel is 15.86m and the steering angle is 3.85 degrees.

From Fig.8, it can be obtained that when the rate of rear velocity change $\dot{\lambda}$=0.01 and the vehicle steers $\theta$=60°, the steering angle $\beta$ of rear part is about 5°. According to the simulations of front wheels, the driving forces of rear wheels at different steering angle $\beta$ of rear part are shown in Fig.13.

![Fig.13 Driving forces of rear wheels](image)

From Fig.13, the rear wheels will complete steering in slid state in the process of steering. When the steering angle $\theta$=0°, the driving force difference between two wheels of front part is 0. The driving force is mainly used to overcoming rolling resistance. And when the steering angle increases, the driving force will be different to overcome ground steering resistance.

5 Conclusions

In this paper, skid steering mode is applied in articulated vehicle of which wheels can be controlled independently to replace the hydraulic steering mode. Based on the kinematic and dynamic model, the characteristics of articulated vehicle with skid steering mode are studied. The main conclusions of this paper are as following.

(1) The hydraulic steering of articulated vehicles could be replaced by skid steering, which can realize the steering at a certain velocity $v$. In the skid steering, the steering angle $\theta$ of articulated vehicle can be enlarged and the turning radius $R$ can be decreased if the body structure of the articulated vehicle permitted.

(2) The steering palstance of the articulated vehicle is related to the vehicle velocity $v$ and the rate of rear wheels velocity change, $\dot{\lambda}$. The steering process of the vehicle can be controlled by the rate of velocity change, $\dot{\lambda}$.

(3) In the process of skid steering, the inside wheel will set in slip steering state earlier than outside wheel.

(4) When the wheel velocity of rear part changes in a linear fashion, the steering angle of the front part is much larger than that of the rear part, which is likely to impact the driver in the steering process. Therefore, it is necessary to make further optimization on the way of the velocity change in the future researches.

References


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