Analysis on the high-speed permanent magnet synchronous motor for FCEV air compressor

Ji-Hwan Choi¹, Seung-Yong Park¹
¹Hyundai MOBIS Electric power engineering team, jihwan@mobis.co.kr

Abstract
The PM motors are considered to be a very promising design alternative for high-speed applications such as compressors, vacuum pumps, turbine generators, flywheel energy-storage systems, drilling tools, friction welding units, etc. This paper presents the electromagnetic field analysis of high-speed permanent magnet synchronous motor (PMSM) for FCEV air compressor using analytical method. The magnetic field by PM and winding current considering slotting effect are presented. Based on field solutions, flux density, unbalanced magnetic force, and design parameters such as back-EMF are derived and compared with FEM results.

Keywords: FCEV, high-speed PM motor, magnetic field analysis

1 Introduction
High-speed machines have been extensively studied with a greater interest over the recent years because of their advantages that include higher power density, increased reliability, and smaller size than conventional medium- or low-speed electrical machines. These high-speed machines are being developed for many industrial applications, such as compressors, vacuum pumps, turbine generators, flywheel energy-storage systems, drilling tools, friction welding units, etc. In all these high-speed industrial units, the application is directly attached to the shaft of the electrical machine and the need for a gearbox. In fact, both the speed and the power of a high-speed machine are controlled by a frequency converter. The development of power electronics enables electric machines to be well operated as high-speed and high-power [1]-[3].

Switched reluctance motors (SRMs), induction machine (IM), and permanent magnet (PM) machines, are suitable for high-speed operation, especially when variable speed required [3]. The PM motors are considered to be a very promising design alternative for high-speed applications since they have better power factor, better drive performance, and greater efficiency with low power loss than IMs and SRMs [1]-[3][4]. Especially, by using retaining sleeve, the rotor of PM motor is protected against the centrifugal force caused by high rotation speed.

Generally finite element method (FEM) or analytical method is used for magnetic field analysis of PM motors. The FEM is more accurate than analytical method and can easily consider nonlinearity such as magnetic saturation effect. On the other hand, it takes a long time to analysis the electromagnetic characteristics. In the analytical method, it is technically difficult to consider nonlinearity. However, magnetic field analysis results are obtained rapidly using analytical method. Therefore, analytical method can be used for initial design of PM motor or design of PM motor without magnetic saturation.

This paper presents the electromagnetic field analysis of high-speed permanent magnet synchronous motor (PMSM) for FCEV air
compressor using analytical method. The magnetic field by PM and winding current considering slotting effect are presented. Based on field solutions, flux density, unbalanced magnetic force, and design parameters such as back-EMF are derived and compared with FEM results.

2 Magnetic field analysis of high-speed PMSM

2.1 Model and Assumptions

Figure 1 shows the structure of high-speed PMSM with concentrated winding, and Figure 2 shows the simplified analytical model for the prediction of magnetic field distributions produced by PMs and stator winding currents obtained from assumptions below:

- The permeability of the stator core is infinite, \( \mu = \infty \).
- The relative permeability of the rotor shaft is \( \mu_{r2} \).
- The relative permeability of the PM is unity, \( \mu = 1 \).
- The current is distributed in an infinitesimal thin sheet at \( r = R_i \).

By assumptions stated above, the analysis of magnetic field due to PMs is confined to three regions represented as air (I), magnet (II) and shaft regions (III).

Here \( R_o, R_i, \) and \( R_e \) represent inner and outer radius of the PMs and inner radius of the stator, respectively. \( \theta \) and \( \theta_i \) are the angular position referred to the rotor and the stator, respectively. The relationship between them is \( \theta = \theta_i + \omega t_i \).

2.2 Magnetic fields produced with permanent magnets

The parallel magnetization \( \mathbf{M} \) can be divided into radial and circumferential component in terms of a vector summation. So, the parallel magnetization \( \mathbf{M} \) is expressed as

\[
\mathbf{M} = \sum_{n=1,3,5,\ldots}^\infty \left[ M_n \cos(np\theta) \mathbf{r} + M_{n0} \sin(np\theta) \mathbf{\theta} \right]
\]

where \( n \) and \( p \) are the \( n^{th} \)-order harmonics and pole-pairs, respectively. \( M_n \) and \( M_{n0} \) are \( n \)-order radial and circumferential Fourier component of the parallel magnetization \( \mathbf{M} \), respectively. Figure 3 shows the magnetization distribution of parallel magnetized 2-pole PM.

As shown in Figure 2(a), since there is no free current in the magnet region, \( \nabla \times \mathbf{H} = 0 \). So, \( \nabla \times \mathbf{B} = \mu_0 \nabla \times \mathbf{M} \). The magnetic vector potential \( \mathbf{A} \) is defined as \( \nabla \times \mathbf{A} = \mathbf{B} \). By the geometry of the rotary machine, the vector potential has only the \( z \)-axial-components. Therefore, the Poisson's equation, in terms of the Coulomb gauge, \( \nabla \times \mathbf{A} = 0 \), is given by
\( \nabla^2 A_{n\text{III}} = 0 \) in the air-gap/shaft regions

\( \nabla^2 A_{\text{II}} = -\nu_0 (\nabla \times \mathbf{M}) \) in the PM regions

Since \( A \) is \( A_{n0} \sin(n \phi \theta) \), equation (2) can be rewritten by

\[
\frac{\partial^2}{\partial r^2} A_{\text{II}} + \frac{1}{r} \frac{\partial}{\partial r} A_{\text{II}} + \frac{A_{\text{II}}}{r^2} = -\frac{\nu_0 \mu_0}{r} \mathbf{M}_n
\]

(3)

where \( \mathbf{M}_n = -(M_n + \text{M}_{0n}/n\pi) \). The homogenous solution of equation (3) is the same as the solution in the air-gap/shaft regions of equation (2), and general solution, which is the sum of homogenous solution and particular solution of equation (3), is the same as solution in the PM regions of equation (2). As a consequence, the solutions for magnetic vector potential of each region shown in figure 2 (a) are expressed as

\[
A_{\text{II}}^{\text{III}} = \sum_{n=1, \text{odd}}^{-\infty} \left[ A_{n0}^{I\text{II}} r^{n\pi} + B_{n0}^{I\text{II}} r^{-n\pi} \right]
\]

(4)

\[
A_{n0}^{\text{II}} = \sum_{n=1, \text{odd}}^{-\infty} A_{n0}^{I\text{II}} + B_{n0}^{I\text{II}} r^{-n\pi} + \frac{\nu_0 \mu_0}{n\pi} - M_n
\]

(5)

\[
A_{n0}^{\text{II}} = \sum_{n=1, \text{odd}}^{-\infty} A_{n0}^{I\text{II}} + B_{n0}^{I\text{II}} r^{-n\pi} + \frac{\nu_0 \mu_0}{n\pi} - M_n
\]

(6)

As mentioned in previous chapter high-speed PM rotor has 2-pole, for the definition of the magnetic vector potential, \( \nabla \times \mathbf{A} = \mathbf{B} \), the analytical solutions for the radial and circumferential flux density due to PM in each region can be obtained as

\[
B_{n0}^{I\text{III}} = \sum_{n=1, \text{odd}}^{-\infty} \left( A_{n0}^{I\text{III}} + B_{n0}^{I\text{III}} r^{-2} \right) \cos \theta
\]

(6.1)

\[
B_{n0}^{\text{II}} = \sum_{n=1, \text{odd}}^{-\infty} \left( A_{n0}^{\text{II}} + B_{n0}^{\text{II}} r^{-2} + \frac{\nu_0 M_n}{2} \ln r \right) \cos \theta
\]

(6.2)

\[
B_{n0}^{I\text{III}} = -\sum_{n=1, \text{odd}}^{-\infty} \left( A_{n0}^{I\text{III}} - B_{n0}^{I\text{III}} r^{-2} \right) \sin \theta
\]

(6.3)

\[
B_{n0}^{\text{II}} = -\sum_{n=1, \text{odd}}^{-\infty} \left( A_{n0}^{\text{II}} - B_{n0}^{\text{II}} r^{-2} + \frac{\nu_0 M_n}{2} \ln r \right) \sin \theta
\]

(6.4)

where undefined coefficients \( A_{n,\text{III}}^{I,\text{III}} - B_{n,\text{III}}^{I,\text{III}} \) can be calculated using boundary conditions. In order to determine the six unfixed coefficients, six boundary conditions are required as follows:

\[
B_{n0}^{I}(R_r, \theta) = B_{n0}^{I\text{III}}(R_r, \theta)
\]

(7.1)

\[
B_{n0}^{I}(R_r, \theta) = B_{n0}^{I\text{III}}(R_r, \theta)
\]

(7.2)

\[
B_{n0}^{I}(R_r, \theta) = 0
\]

(7.3)

\[
B_{n0}^{I}(R_r, \theta) - B_{n0}^{I\text{III}}(R_r, \theta) = \mu_n M_{n0} \sin \theta
\]

(7.4)

\[
B_{n0}^{I}(R_r, \theta) - B_{n0}^{I\text{III}}(R_r, \theta) / \mu_n = -\mu_n M_{n0} \sin \theta
\]

(7.5)

\[
A_{n0}^{I\text{III}}(R_r = 0, \theta) = 0
\]

(7.6)

Equation (7.1) and equation (7.2) are obtained by the continuity of the radial component of flux density \( \mathbf{B} \) at each boundary surface. Equation (7.3), equation (7.4) and equation (7.5) are obtained by the continuity of the circumferential component of magnetic field \( \mathbf{H} \). In addition, equation (7.3) is obtained by the assumptions that the permeability of the stator core is infinite, and circumferential component of \( \mathbf{M} \) is considered in boundary condition as shown in equation (7.4) and equation (7.5). Equation (7.5) is obtained by the assumption that the magnetic vector potential is non-existent at the origin.

By substituting equation (6) ~ (7), this paper obtains the matrix as equation (8), and undefined coefficients \( A_{n,\text{III}}^{I,\text{III}} - B_{n,\text{III}}^{I,\text{III}} \) can be obtained:

\[
\begin{bmatrix}
1 & -1 & 0 & R_{r0}^{-2} & -R_{r0}^{-2} & 0 \\
0 & 1 & -1 & 0 & R_{r0}^{-2} & -R_{r0}^{-2} \\
1 & 0 & 0 & -R_{r0}^{-2} & 0 & 0 \\
1 & -1 & 0 & -R_{r0}^{-2} & R_{r0}^{-2} & 0 \\
0 & 1 & -1/\mu_n & 0 & -R_{r0}^{-2} & R_{r0}^{-2}/\mu_n \\
0 & 0 & 0 & 0 & 1 & B_{n0}^{I\text{III}}
\end{bmatrix}
\]

(8)

Finally, the radial and circumferential flux density can be determined at any point in air-gap, magnet and rotor shaft using obtained coefficients and equation (6.1) ~ (6.4).
2.3 Magnetic fields produced with stator winding currents

The Fourier series expansion for current density of phase a is obtained as

\[ J_a = \sum_{n=1, odd}^{\infty} I_n i_a \cos(n p \theta) \]  

(9)

where \( i_a \) represents current which flows through phase a. \( I_n \) is the \( n \)th-order Fourier series coefficient for the current density distribution and is expressed as

\[ I_n = \frac{2N_{np}}{nh_{p} \pi} \left[ \sin \left( n p \frac{b_r}{2R_r} \right) + \sin \left( n p \frac{\pi}{p} \frac{b_r}{2R_r} \right) \right] \]  

(10)

where \( N_{np} \) and \( b_r \) are the number of turns per pole per phase and the width of a stator slot opening. Since 3-phase stator windings are wound 2\pi/3 electrical radians apart in space, the Fourier series expansions for phase b and c are obtained as

\[ J_b = \sum_{n=1, odd}^{\infty} I_n i_b \cos \left( n p \left( \theta - \frac{2\pi}{3p} \right) \right) \]  

(11)

\[ J_c = \sum_{n=1, odd}^{\infty} I_n i_c \cos \left( n p \left( \theta - \frac{4\pi}{3p} \right) \right) \]  

(12)

where \( i_b \) and \( i_c \) represent current which flows through phase b and c, respectively. As a consequence, the current density \( J \) for stator windings is expressed as

\[ J = J_a + J_b + J_c \]  

(13.1)

\[ J_a = \sum_{n=1, odd}^{\infty} I_n i_a \cos(n p \theta) \]  

(13.2)

\[ J_b = \sum_{n=1, odd}^{\infty} I_n i_b \cos \left( n p \left( \theta - \frac{2\pi}{3p} \right) \right) \]  

(13.3)

\[ J_c = \sum_{n=1, odd}^{\infty} I_n i_c \cos \left( n p \left( \theta - \frac{4\pi}{3p} \right) \right) \]  

(13.4)

Figure 4 shows the current density distribution of current sheet model obtained from equation (13). Through the similar steps as in the PM case, the governing equation due to stator winding currents is represented by Laplace’s equation as follows:

\[ \nabla^2 \mathbf{A}_{\text{st}} = 0 \]  

(14)

The radial and circumferential flux density due to stator winding currents, \( B_{\text{cm}} \) and \( B_{\text{ch}} \) are obtained as

\[ B_{\text{cm}}^{\prime} = \sum_{n=1, odd}^{\infty} n p \left\{ C_{n}^{\prime} r^{n-p-1} + D_{n}^{\prime} r^{-n-p-1} \right\} \sin(n p \theta) \]  

(15.1)

\[ B_{\text{cm}}^{\prime} = \sum_{n=1, odd}^{\infty} n p \left\{ C_{n}^{\prime} r^{n-p-1} + D_{n}^{\prime} r^{-n-p-1} \right\} \cos(n p \theta) \]  

(15.2)

\[ B_{\text{cm}}^{\prime} = - \sum_{n=1, odd}^{\infty} n p \left\{ C_{n}^{\prime} r^{n-p-1} - D_{n}^{\prime} r^{-n-p-1} \right\} \cos(n p \theta) \]  

(15.3)

\[ B_{\text{cm}}^{\prime} = - \sum_{n=1, odd}^{\infty} n p \left\{ C_{n}^{\prime} r^{n-p-1} - D_{n}^{\prime} r^{-n-p-1} \right\} \cos(n p \theta) \]  

(15.4)

In order to determine the undefined coefficients in equation (15.1) – (15.4), four boundary conditions are required as follows:

\[ B_{\text{cm}}^{\prime} (R_i, \theta) = B_{\text{cm}}^{\prime} (R_i, \theta) \]  

(16.1)

\[ B_{\text{ch}}^{\prime} (R_i, \theta) = - \mu_{r} J_{a,b,c} \]  

(16.2)

\[ \mu_{r} B_{\text{ch}}^{\prime} (R_i, \theta) = B_{\text{ch}}^{\prime} (R_i, \theta) \]  

(16.3)

\[ A_{\text{cm}}^{\prime} (R_i = 0, \theta) = 0 \]  

(16.4)

Equation (16.1) is obtained by the continuity of the radial component of flux density, and equation (16.2) and equation (16.3) are obtained by the continuity of the circumferential magnetic fields.

EVS28 International Electric Vehicle Symposium and Exhibition 4
Especially, equation (16.2) is obtained by the assumption that the current is distributed in an infinitesimal thin sheet at \( r = R_a \). Equation (16.4) is obtained by the assumption that the magnetic vector potential is non-existent at the origin.

By substituting equation (15.1) \~ (15.4), this paper obtains the matrix as equation (17), and undefined coefficients \( C_n^{III} \~ D_n^{III} \) can be obtained by solving the matrix.

\[
\begin{bmatrix}
R_0^{mp} & -R_0^{mp} & -R_0^{mp} & 0 \\
0 & -R_0^{mp} & -R_0^{mp} & 0 \\
-\mu_0 R_0^{mp} & 0 & -\mu_0 R_0^{mp} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
C_p \\
C_p \\
\mu_0 C_p \\
\mu_0 C_p
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
-\frac{\mu_0}{\mu_p} \mu_{p0} \rho_{p0} \\
0
\end{bmatrix}
\]

Finally, the radial and circumferential flux density can be determined at any point in air-gap and rotor shaft using obtained coefficients and equation (15.1) \~ (15.4).

### 2.4 2-D Permeance function for consideration of slotting effect

Slotting affects the magnitude and shape of magnetic flux density in airgap and magnet. The equivalent air gap is equal to the physical air gap multiplied by the so-called “Carter factor coefficient.” This concept was well accepted for the design of electric machines. The 2-D permeance function can be expressed as [5]

\[
\lambda_r (r, \theta) = \Lambda_0 (r) + \sum_{n=1}^{\infty} \Lambda'_r (r) \cos \mu Q_s (\theta_s + \theta_m) \quad (18)
\]

\[
\lambda_c (r, \theta) = \sum_{n=1}^{\infty} \Lambda'_c (r) \cos \mu Q_s (\theta_s + \theta_m) \quad (19)
\]

where \( \lambda_r \) and \( \lambda_c \) are the 2-D permeance function applied to radial and circumferential flux density due to PM and stator winding currents in order to consider slotting effects, respectively. \( Q_s \) is the number of slots. \( \theta_m \) is determined by the winding pitch, i.e., the value of \( \theta_m \) have 0 and \( \pi/Q_s \) for the case when the winding pitch is an odd and an even integer of the slot pitch, respectively. \( \Lambda_0 \) and \( \Lambda'_r \) are given in [6]. The camper coefficient is given by

\[
K_r = \frac{\tau_r}{\tau_r - \gamma g} \quad (20)
\]

where \( g' \) is the effective air-gap given by \( (R_r - R_i) \). \( \gamma \) is given by

\[
\gamma = \frac{4}{\pi} \left[ \frac{b_{r0}}{2g} \tan^{-1} \left( \frac{b_{r0}}{2g} \right) - \ln \left( 1 + \left( \frac{b_{r0}}{2g} \right)^2 \right) \right] \quad (21)
\]

Using equation (18) and (19), the radial and circumferential flux density due to PM considering slotting effects in the air-gap regions can be obtained as

\[
B'_{cm \_slotted} (r, \theta) = B'_{cm} (r, \theta) \rho_{c'} (r, \theta) \quad (22)
\]

\[
B'_{cm \_stood} (r, \theta) = B'_{cm} (r, \theta) + B'_{cm} (r, \theta) \rho_{c'} (r, \theta) \quad (23)
\]

The radial and circumferential flux density due to stator winding currents considering slotting effects in the air-gap regions can also be obtained as

\[
B'_c (r, \theta) = B'_c (r, \theta) \rho_{c'} (r, \theta) \quad (24)
\]

\[
B'_{c0 \_slotted} (r, \theta) = B'_{c0} (r, \theta) + B'_{c0} (r, \theta) \rho_{c'} (r, \theta) \quad (25)
\]

### 2.5 Flux Linkages due to PM and Stator Winding Currents

The flux linkages due to PM and stator winding currents are required to estimate back-EMF and a winding inductance. Stator slotting reduces effective flux density according to average value of relative permeance \( \Lambda_0 \). Therefore, the flux density due to PM and stator winding currents given in equation (6.1) and (15.1) can be rewritten as

\[
B_m^i = \Lambda_0 \sum_{n=1, odd}^{\infty} B_n \cos (np\theta) \quad (26)
\]
\[ B'_{cm, n} = \sum_{n=1, even}^{n \infty} B_n \sin(n \theta_n) \]  \hspace{1cm} (27)

where \( B_n \) and \( B_{cm, n} \) are the function for radial position \( r \).

### 2.5.1 Flux Linkages due to PM

The flux linkage due to PM can be calculated by

\[ \psi_{PM} = R_l l_s \int_{-\alpha_s/2}^{\alpha_s/2} B'_m d\theta_s \]  \hspace{1cm} (28)

where \( l_s \) and \( \alpha_s \) are the stator stack length and coil pitch, respectively. By substituting equation (26) for equation (28),

\[ \psi_{PM} = N_{pp} \Lambda_n R_l l_s \sum_{n=1, odd}^{n \infty} B_n \int_{-\alpha_s/2}^{\alpha_s/2} \cos(n \theta_s - \alpha_s t) d\theta \]

\[ = \frac{N_{pp} \Lambda_n R_l l_s}{\mu_0} \sum_{n=1, odd}^{n \infty} \frac{B_n}{n} \left[ \cos \left( \frac{n \alpha_s - \alpha_f}{2} \right) + \cos \left( \frac{n \alpha_s + \alpha_f}{2} \right) \right] \]

\[ = \frac{2N_{pp} \Lambda_n R_l l_s}{\mu_0} \sum_{n=1, odd}^{n \infty} \frac{B_n}{n} \cos \left( \frac{n \alpha_s}{2} \right) \cos(n \theta_s t) \]  \hspace{1cm} (29)

where \( N_{pp} \) is turns per phase per pole. So, flux linkage per phase due to PM is given by

\[ \psi_{PM} = 2N_{pp} \Lambda_n R_l l_s \sum_{n=1, odd}^{n \infty} \frac{B_n}{n} \left[ \frac{n \alpha_s}{2} \right] \cos(n \theta_s t) \]  \hspace{1cm} (30)

### 2.5.2 Flux Linkages due to Stator Winding Currents

Through the similar steps as in the flux linkages due to PM, the flux linkage due to stator winding currents can be calculated from

\[ \psi_{currents} = R_l l_s \int_{-\alpha_s/2}^{\alpha_s/2} B'_m d\theta_s \]  \hspace{1cm} (31)

By substituting equation (27) for equation (31), flux linkage per phase due to stator winding currents is given by

\[ \psi_{currents} = N_{pp} \Lambda_n R_l l_s \sum_{n=1, odd}^{n \infty} B_n \int_{-\alpha_s/2}^{\alpha_s/2} \sin(n \theta_s) d\theta_s \]

\[ = -N_{pp} \Lambda_n R_l l_s \sum_{n=1, odd}^{n \infty} \frac{B_n}{n} \left[ \cos \left( \frac{n \alpha_s}{2} \right) + \cos \left( \frac{n \alpha_s}{2} \right) \right] \]

\[ = -2N_{pp} \Lambda_n R_l l_s \sum_{n=1, odd}^{n \infty} \frac{B_n}{n} \cos \left( \frac{n \alpha_s}{2} \right) \]  \hspace{1cm} (32)

### 2.6 Unbalanced magnetic force and design parameter estimation

#### 2.6.1 Unbalanced magnetic force

The unbalanced magnetic force is the resultant global magnetic force that acts on the rotor due to an asymmetric magnetic field distribution in the air-gap. It can be calculated either analytically or by finite-element analysis using Maxwell’s stress tensor method. This unbalanced radial force will lead the great vibration on the stator.

Using estimated radial and circumferential flux density, unbalanced magnetic force \( F_r \) and radial force density \( f_r \) can be calculated as

\[ f_r = \frac{\left( B'_{r, cm, slotted} + B'_{r, cm, slotted} \right)^2 - \left( B'_{r, cm, slotted} + B'_{r, cm, slotted} \right)^2}{2\mu_0} \]  \hspace{1cm} (33)

\[ F_r = R_l \int_0^{\pi} f_r d\theta \]

\[ = R_l \frac{\left( B'_{r, cm, slotted} + B'_{r, cm, slotted} \right)^2 - \left( B'_{r, cm, slotted} + B'_{r, cm, slotted} \right)^2}{2\mu_0} \]  \hspace{1cm} (34)

Flux density components due to stator winding current, \( B'_{cm, slotted} \) and \( B'_{cm, slotted} \) are neglected in no-load state.

#### 2.6.2 Back-EMF

The back-EMF induced in a stator winding due to magnet flux crossing the air gap is calculated as

\[ e_{back} = -\frac{d\psi_{PM}}{dt} \]  \hspace{1cm} (35)

By substituting equation (30) for equation (35) the back-EMF per phase is obtained as
Back-EMF constant is expressed as

\[ k_\text{e} = \max \left( \frac{e_\text{lock}}{\omega} \right) \]  

(37)

2.6.3 Inductance

Self inductance \( L_{\text{self}} \) is the sum of air-gap inductance \( L_a \) and end-winding inductance \( L_{\text{end}} \). The air-gap inductance \( L_a \) can be calculated from equation (31) by the formula

\[ L_a = -2N_{\text{app}} \lambda_0 R_L \sum_{n=1, \text{odd}} B_{n, \alpha} \cos \left( \frac{np \alpha}{2} \right) \]  

(38)

End-winding inductance is difficult to calculate accurately with simple formulas because the conformation of the end-windings is complex and difficult to characterize mathematically in simple terms. Fortunately, end winding inductance is generally quite small, because end winding of concentrated winding machines is relatively short. Therefore, end winding inductance is ignored in this paper.

The mutual inductance of \( j \)-phase due to the current in the \( k \)-phase is as follows:

\[ M_{\text{mutual}_{jk}} = \left. \frac{\psi_{a-j}}{i_k} \right|_{i_k=0} \]  

(39)

As shown in figure 1, 3-slot/2-pole concentrated winding machine has 3-phase winding separated as 120°, but 6-slot/2-pole concentrated winding machine has 3-phase winding separated as 60°. The mutual inductance of 3-slot/2-pole and 6-slot/2-pole concentrated winding machines are expressed by the ratio of the mutual and self inductances as follows

\[ M_{\text{mutual}} = \cos 120^\circ \frac{L_{\text{self}}}{L_{\text{self}}} = -\frac{1}{2} L_{\text{self}} \]  

(40)

\[ M_{\text{mutual}} = \cos 60^\circ \frac{L_{\text{self}}}{L_{\text{self}}} = \frac{1}{2} L_{\text{self}} \]  

(41)

Table 1: Specification of initial analysis model for high-speed PMSM with concentrated winding

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole number</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>Slot opening</td>
<td>mm</td>
<td>3 ~ 8</td>
</tr>
<tr>
<td>Rotor outer diameter</td>
<td>mm</td>
<td>37</td>
</tr>
<tr>
<td>PM thickness</td>
<td>mm</td>
<td>8.5</td>
</tr>
<tr>
<td>Sleeve thickness</td>
<td>mm</td>
<td>2</td>
</tr>
<tr>
<td>Shaft outer diameter</td>
<td>mm</td>
<td>16</td>
</tr>
<tr>
<td>Stack length</td>
<td>mm</td>
<td>84</td>
</tr>
<tr>
<td>Airgap thickness</td>
<td>mm</td>
<td>1</td>
</tr>
<tr>
<td>Stator outer diameter</td>
<td>mm</td>
<td>90</td>
</tr>
<tr>
<td>Slot number</td>
<td>-</td>
<td>3-slot 6-slot</td>
</tr>
<tr>
<td>Turns per phase</td>
<td>-</td>
<td>20 36</td>
</tr>
</tbody>
</table>

Under the condition of balanced 3-phase current, the flux linking the \( a \)-phase winding can be expressed as

\[ \psi_a = \left( L_{\text{self}} + L_a \right) i_a + M_{\text{mutual}} h_b + M_{\text{mutual}} h_c \]

\[ = \left( \frac{3}{2} L_{\text{self}} + L_a \right) i_a = L_i i_a \]  

(42)

\[ \psi_a = \left( L_{\text{self}} + L_a \right) i_a + M_{\text{mutual}} h_b + M_{\text{mutual}} h_c \]

\[ = \left( \frac{1}{2} L_{\text{self}} + L_a \right) i_a = L_i i_a \]  

(43)

Equation (42) is for 3-slot/2-pole, and equation (43) is for 6-slot/2-pole. Here, \( L_{\text{ref}} \) and \( L_s \) are the leakage inductance and synchronous inductance, respectively.

3 Magnetic field analysis results

Table 1 presents the specification of initial analysis model for high-speed PMSM. The stator outer diameter and airgap thickness were determined to have same size with previously manufactured distributed winding machine. Also, solid wire is used instead of stranded wire for using automatic winding machine. Minimum slot opening is presented as 3 mm for automatic winding.
3.1 Magnetic field characteristic due to PM

Figure 5 – 8 shows the comparison of airgap flux density due to PM with 2-D FEM and analytical result by equation (22) and (23). Figure 5 and 6 show the radial and circumferential components of airgap flux density distribution for the 3-slot/2-pole machine due to PM when slot opening is 3 mm and 6 mm. Figure 7 and 8 show the radial and circumferential component of airgap flux density distribution for the 6-slot/2-pole machine due to PM with slot opening of 3 mm and 6 mm. Figure 5 – 8 shows that stator slotting makes the local variation of airgap flux density. At position of slot opening, while circumferential flux density increases, radial flux density decreases. This slotting effect on the airgap flux density increases with increasing slot opening.

Analytical results for airgap flux density due to PM show the good agreement with 2-D FEM results. However, difference with analytical result and 2-FEM result increase with increasing of slot opening.

3.2 Magnetic field characteristics due to winding current

Figure 9 – 12 show the comparison of airgap flux density due to stator winding current with 2-D FEM and analytical result by equation (24) and (25). The stator winding current is 20 A. Figure 9 and 10 show the radial and circumferential component of airgap flux density distribution for the 3-slot/2-pole machine due to PM when slot opening is 3 mm and 6 mm.

Figure 11 – 12 show the radial and circumferential component of airgap flux density distribution for the 6-slot/2-pole machine due to PM with slot opening of 3 mm and 6 mm. Figure 9 – 12 show that value of airgap flux density due to stator winding current decrease with increasing slot opening. Analytical results for airgap flux density due to stator winding current show the good agreement with 2-D FEM results.
Figure 7: Airgap flux density distribution for the 6-slot/2-pole machine due to PM with 3 mm slot opening: (a) radial component and (b) circumferential component.

Figure 8: Airgap flux density distribution for the 6-slot/2-pole machine due to stator winding current (I=20A) with 3 mm slot opening: (a) radial component and (b) circumferential component.

Figure 9: Airgap flux density distribution for the 3-slot/2-pole machine due to stator winding current (I=20A) with 6 mm slot opening: (a) radial component and (b) circumferential component.

Figure 10: Airgap flux density distribution for the 3-slot/2-pole machine due to stator winding current (I=20A) with 6 mm slot opening: (a) radial component and (b) circumferential component.
3.3 Unbalanced magnetic force characteristics

Figure 13 and 14 show the comparison of the radial force density characteristic with analytical and 2-FEM results for the 3-slot/2-pole and 6-slot/2-pole machine. Figure 13 shows the radial force density for the 3-slot/2-pole machine when slot opening is 3mm and 6mm. Figure 14 shows the radial force density for the 6-slot/2-pole machine when slot opening is 3mm and 6mm. The radial force density distribution of 3-slot/2-pole machine is asymmetric. On the other hand, the radial force density distribution of 6-slot/2-pole machine is almost symmetric. Moreover, asymmetric characteristic of radial force density worsens with increasing of slot opening.

This asymmetric characteristic causes the unbalanced magnetic force as shown in figure 15 and 16. These figures show the unbalanced magnetic force for the 3-slot/2-pole machine and 6-slot/2-pole machine according to slot opening in no-load and load condition, respectively.
From these figures, it is noted that unbalanced magnetic force increase with increasing of slot opening. Also, unbalanced magnetic force in load condition is larger than in no-load condition.

Unbalanced magnetic force of the 3-slot/2-pole machine is more than 50 N in the load condition. On the other hand, unbalanced magnetic force of the 6-slot/2-pole is below 0.01 N. Since unbalanced magnetic force become a reason of noise and vibration, the 3-slot/2-pole machine is inappropriate in aspect of noise and vibration.

### 3.4 Back-EMF characteristic

Figure 17 and 18 show the back-EMF characteristic for the 3-slot/2-pole machine and 6-slot/2-pole machine at 30000 rpm. Figure 17(a) and figure 18(a) are phase back-EMF wave form with 3mm slot opening. The back-EMF forms of both machines are clearly sinusoidal. Figure 17(b) and figure 18(b) are pick value of phase back-EMF according to various slot opening. The back-EMF value decreases with increasing slot opening as shown in these figures.
4 Result and discussion

This paper presented the electromagnetic field analysis of high-speed PMSM for FCEV air compressor using analytical method. The magnetic field by PM and winding current considering slotting effect were presented. Based on field solutions, flux density, unbalanced magnetic force, and design parameter such as back-EMF were derived and compared with FEM results. Analytical results, such as airgap flux density, back-EMF, and unbalanced magnetic force, showed the good agreement with 2-D FEM results.

References


Authors

Ji-Hwan Choi was born in Korea in 1983. He received the M.S and Ph.D. degrees in electrical engineering from the Chungnam national university, Korea, in 2011 and 2014, respectively. He is currently an Principal research engineer with Hyundai MOBIS.