Electromagnetic Equivalent Circle Modelling of Interior Permanent Magnet Synchronous Machine Using Modelica

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Abstract
This paper proposes an electromagnetic equivalent circle (EMEC) model to evaluate and analyze the performance of interior permanent magnet synchronous machine (IPMSM). This model is implemented with Modelica language and it provides magnetic field simulation which is not included in PMSM model from Modelica standard library (MSL). Moreover, this model simulates the variable working states of permanent magnet (PM) instead of a constant flux source. The electric part of EMEC model is similar to that in MSL, except that the calculation of induced electromotive force (EMF) is divided into motional EMF and back EMF. As for magnetic part, this physical model considers the magnetic fields distribute evenly in several lines and takes magnetomotive force (MMF) as potential variable and the magnetic flux as flow variable. This paper doesn’t adopt lumped inductance parameters, and the permeances of icon core, air gap and magnets are based solely on geometrical data and material parameters. The model takes into account local saturation of individual stator teeth by considering the permeance of iron core is nonlinear. A dq framework is utilized to determine the directions of electric and magnetic space vectors. The magnetic flux produced by magnets is oriented in d-axis and its density is constant over one pole pitch. And the working points of magnets follow its demagnetization curve. The MMF produced by a balanced three-phase sinusoidal winding current system is supposed to be circumferential traveling wave with sinusoidal waveform, and only the d-axis component of its resulting magnetic flux crosses magnets. Finally, EMEC model is validated through comparisons with a finite element model confronting the selected analytical waveforms, as well as the electromagnetic torques, and it is used in a system model to help developing motor control algorithms.

Keywords: Interior permanent magnet synchronous machine(IPMSM), Electromagnetic equivalent circle(EMEC), Magnetic saturation, Modelica

1 Introduction
Electric ones from all kinds of new clear energy vehicles are most widely applied to practice, while electric motor system is one of their key components. There are several types of electric motors used for driving, and Interior Permanent-magnet synchronous machines (IPMSMs) are of great interest in high performance applications due to their high power density, reliability, very good efficiency, low noise and their reduced maintenance costs. Moreover, the magnets’ cost reduction and
improvements achieved in magnetic material properties contribute to PMSM market’s expansion[1].

Modelling and simulation are usually used to analyze and optimize vehicle performances, due to their short development cycle and low cost. However, IPMSM system involves multi-physics domains such as magnetical, mechanical and electrical, and has complex structure and behaviour. Its model is nonlinear and multi-physics coupled.

To facilitate the design and analysis of PM machines, a precise computation of the magnetic field distribution in the different machine regions is required[2]. Generally, accurate modelling of electric machines requires the use of finite-element method. However, FE analysis is too time consuming, and the use of MEC models is often preferred because they allow the exploration of the whole search space of solutions while reducing the pre-design stages duration[3].

Conventional MEC models of IPMSM only describe the magnetic field distribution in airgap, while haven’t considered equivalent electric circles of stator windings and combined them with equivalent magnetic circles. Bödrich introduces a simple EMEC model of actuator motors[4], and Kral presents an EMEC model of PMSM[5]. But both of them use lumped inductance parameters and considered them to be constant. Due to excessive simplification, the EMCM model lacked the ability to accurately predict the flux saturation and machine performances.

Sjöstedt[6] concludes four realization levels of the model of continuous-time physical systems, which are physical modelling models, constraint models, continuous causal models and discretized models with solver, and he states that physical modelling is more suitable for physical system due to its more flexibility and extensibility. Conventional MEC models usually implemented in continuous causal ones[7], thus this paper chooses Modelica language to model MEC model of IPMSM. Modelica is a physical modeling language that allows specification of mathematical models for the purpose of computer simulation of dynamical systems. Modelica[8] is a free object-oriented modeling language with a textual definition to describe physical systems in a convenient way by differential, algebraic and discrete equations. Modelica models describe topological structure of how components are interconnected and uses the connect statement to connect components, and equations can be associated with the connector. In the connector, flow variables are defined. When two ports are connected, potential variables are set to equal, and flow variables are summed to zero.

In this paper, structure and theory of IPMSM will be analyzed. Then, an EMEC model in dq frame that is able to obtain the information of both electric and magnetic vectors will be presented and implemented with Modelica language. The simulation curves of inductance matrix in d- and q-axis will be discussed by comparison with FEM analysis and closed-loop simulation of IPMSM system at the end of paper.

2 Analysis of Interior Permanent Magnet Synchronous Machine

2.1 Basic Mathematical Model of PMSM

With the following assumptions[9]:

(1) The machine is symmetrical;
(2) Saturation effect of main flux is significant;
(3) Skin-effect and temperature effect are neglected;
(4) Harmonic content of the MMF wave is neglected;
(5) The induced EMF is sinusoidal, and the stator voltages are balanced;
(6) There is no dampener winding on rotor.

The mathematical description of a PMSM in abc frame is given by the following equations:

\[ \psi_{abc} = L_{abc}i_{abc} + \psi_{fabc} \quad (1) \]
\[ u_{abc} = R_{abc}i_{abc} + L_{s} \frac{di_{abc}}{dt} + j\omega_{s}\psi_{fabc} \quad (2) \]
\[ \tau_{c} = P\psi_{f}i_{s} \sin \beta \quad (3) \]

Where \( u_{abc}, i_{abc} \) and \( \psi_{abc} \) are vectors of the stator phase voltages, currents and magnetic fluxes. \( i_{s} \) is the magnitude of \( i_{abc} \). \( \tau_{c} \) is the electrical torque produced by motor. \( \psi_{fabc} \) is the main magnetic flux linkages produced by PM and \( \psi_{f} \) refers to its magnitude. \( \beta \) is the angle between \( \psi_{fabc} \) and \( i_{abc} \). \( P \) is the number of pole pairs. \( \omega_{s} \) is angular velocity which equals to derivative of electrical angle \( \theta \). The connection between electrical angle and actual mechanical one is \( \theta = P \cdot \theta_{mech} \). \( R_{abc} = diag\{R_{a}, R_{b}, R_{c}\} \) and \( L_{abc} \) are the stator resistance and inductance matrices.

\[ L_{abc} = \begin{bmatrix} L_a & M_{ab} & M_{ac} \\ M_{ba} & L_b & M_{bc} \\ M_{ca} & M_{cb} & L_c \end{bmatrix} \quad (4) \]

Where \( L_a, L_b \) and \( L_c \) are position dependent self inductances of the three stator phases, and the other elements are mutual inductances. Coordinate transformations of the original model (1)-(3) often result in simpler motor models that offer advantages in later analysis and model. In the
most common motor drive setting, motor phases are arranged either in a three wire star connection or in a delta connection. In that case phase currents or voltages sum up to zero, and the original three phase model can be reduced to a two phase model with an appropriate transformation. In addition, a rotating transformation is often introduced as well to eliminate position dependence of the fundamental components in the model. The combined transformation involves multiplication of all vector variables with the following transformation matrix:

\[
T = \begin{bmatrix}
N_3 \cos \theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta - \frac{4\pi}{3}) \\
N_2 \sin \theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{4\pi}{3})
\end{bmatrix}
\]

Where \( N_3 \) is the number of series winding turns of each phase in abc reference frame. \( N_2 \) is the number of equivalent series turns of each phase in dq reference frame. Therefore, the variables in dq frame can be transformed from the following equation:

\[
X_{dq} = T \cdot X_{abc}
\]

2.2 Configurations and Fields of PM

According to the magnetization direction of PM with respect to rotor rotation, there are three major variations of the rotor of an IPMSM in which the magnets are located in the rotor interior. As is shown in Figure 1, PM in (a) is tangential magnetized, (b) radial magnetized and (c) mixed magnetized.

![Figure 1: Rotor configuration](image)

3 Proposed Electromagnetic Equivalent Circuit Model

3.1 MEC Model of PMSM in MSL 3.2.1

Electric and MEC PMSM models have been introduced to the Modelica Standard Library (MSL)
in 2004[10] and 2011[5]. Figure 2 shows the MEC model of PMSM in MSL, which consists of stator windings, PM, rotor cage, stator and rotor inertia and loss components. This model introduces magnetic flux tubes to model fields and shows the real physical topological structure and involves electric, magnetic and mechanical domains. Sophisticated loss effects covering friction losses, eddy current core losses, stray load losses and brush losses as well as a consistent thermal concept have been implemented for both the two models. The main difference between the two models is that electric PMSM model uses current, voltage and flux linkage space phasors (vectors) whereas MEC one applies complex vectors for physical representation of the magnetic flux and the magnetic potential difference.

Figure 2: MEC model of PMSM in MSL 3.2.1

Since MEC model of PMSM in MSL is built based on lump inductance parameters (L_d, L_q), it cannot describe machine geometrical structure, PM size and types. Moreover, magnetic saturation of PMSM is not included in the model and can only be modelled by varying inductances with flux.

3.2 Equivalent Electric Circuit

In this paper, the armature side of the motor is modeled as an equivalent electric circuit consisting of an ideal voltage source (EMF) and an armature resistance. According to the EMF mechanism, it can be divided into two parts: motional and back EMF. Motional EMF \( e_0 \) is caused by rotor’s rotation, while back EMF \( e_m \) is produced due to alternation of magnetic linkages across the stator windings. EMF block relates electrical voltage and current to magnetic potential difference and flux due to the induction law. An additional winding factor \( k_w \) adjusts magnetic to electrical values due to simplified modeling and field geometry and can be calculated by means of the rotating field theory from the distribution factor and the short-pitch factor. EMF equations are expressed as:

\[
e_0 = \begin{bmatrix} -k_w \omega_r \psi_d \\ k_w \omega_r \psi_q \\ \end{bmatrix} = k_w \omega_r N_2 \begin{bmatrix} -\psi_d \\ \psi_q \\ \end{bmatrix} \quad (11)
\]

\[
e_m = \frac{d\psi}{dt} = k_w \omega_r N_2 \begin{bmatrix} d\psi_d \\ d\psi_q \\ \end{bmatrix} \quad (12)
\]

With reference to formula (8), (11) and (12), the following equation can be derived:

\[
u_s = R_s i_s + e_m + e_0 \quad (13)
\]

The equivalent electric circuit of PMSM armature is based on the analysis above that the armature has a resistance \( R_s \), motional and back EMF.

Figure 3: d-axle equivalent circle

Figure 4: q-axle equivalent circle
3.3 Magnetic Equivalent Circuit

This paper utilizes magnetic flux tubes in [5] to model magnetic fields, which are magnetic equivalent circuits. That is to say that a section of magnetic circuit is a defined volume inside a magnetic field with homogenous distribution of the magnetic field strength and the magnetic flux density within this region.

3.3.1 Fields Produced by PM Acting Alone

PM is made of hard magnetic materials which are hardly magnetized and demagnetized. It typically exhibits a very wide magnetic hysteresis loop and operates in quadrant III, which is called demagnetized curve. PMs are designed to operate on the linear part of the demagnetized curve, so the curve is simplified and shown in Figure 5.

There are two important quantities associated with the demagnetization curve that are the residual flux density $B_r$ and coercive force $H_c$. As for a given PM, the residual flux holds:

$$\phi_r = B_r A_m = B_r h_m L$$

(14)

Where $A_m$, $h_m$ and $L$ are the cross-sectional area, height and length of PM respectively. PM is equivalent to a constant flux $\phi_r$ source in parallel with a resistor $\Lambda_0$.

The PMs produce a flux which is divided into a component circulating through the airgap and stator and a leakage component via the nonmagnetic hub and the rotor shaft. The former component participates in the energy conversion, while the later one does not participate in electromechanical conversion. Combining the fluxes produced by both armature currents and PMs, the physical models of IPMSM in dq frame are shown in Figure 8 and Figure 9.

3.3.2 Magnetic Equivalent Circuit of IPMSM on Load

The windings positioned in the stator, which are supposed to be symmetrical, produce airgap MMF with square waveform. The airgap MMF can be decomposed into the sum of spatial harmonics using the Fourier series, and the fundamental component $F_s$ is sinusoidal distribution with:

$$F_s = \left[\begin{array}{c} F_{sd} \\ F_{sq} \end{array}\right] = 4 \frac{n_s k_w}{\pi} \cos(\theta_e) \cdot \left[\begin{array}{c} i_{sd} \\ i_{sq} \end{array}\right]$$

(15)

However, integration for non-time variables cannot be realized. In order to avoid integral operation, $F_s$ is equivalent to square waveform $F_{sm}$, and it holds: Airgap MMF fundamental harmonic curve and its equivalent square waveform are shown in Figure 7.
The magnetic resistances of the iron core of rotor and stator yoke are usually small and can be neglected when modelling. Furthermore, the leakage resistance with respect to armature fluxes currents is not taken into consideration. So the resistances of IPMSM contains stator teeth resistance, airgap resistance PM resistance and nonmagnetic hub resistance. Magnetic equivalent circle models in dq frame are obtained from the analysis above, which are shown in Figure 10 and Figure 11.

3.3.3 Reluctances Modelling

Magnetic saturation only happens in stator teeth in most cases and saturation in other regions can be ignored by contrast. Therefore, the resistances of airgap, PM and nonmagnetic hub are considered to be constant, while the resistance of stator teeth changes with flux density. From the view of modelling, the airgap, PMs and nonmagnetic hub are linear flux tube elements while stator teeth are nonlinear one. The calculation of reluctance of flux tube elements is based on their materials, shapes and the direction of flux. In order to determine the length and area of each element, the expected flux direction and its variation are established first. One side of every element is along the axial direction and it is equal to the stack length of the machine. The other side of the area is the element width. The length of each element is taken along the flux direction.

PM is a cubic element and the flux flows through rectangular cross-section. Its reluctance for each magnetic pole \( \frac{1}{\Lambda_0} \) is calculated as follow:

\[
\Lambda_0 = \mu_m \mu_0 \frac{A_m}{\varepsilon_m} = \mu_m \mu_0 \frac{r_m L}{\varepsilon_m}
\]

Where, \( \varepsilon_m \) is the thickness of PM; \( \mu_m \) is relative permeability of PM and \( \mu_0 \) is the absolute permeability of vacuum.

Both airgap and nonmagnetic hub are hollow cylindrical elements with axial magnetic flux, and their thicknesses keep constant at the arbitrary angle. Their reluctances for each magnetic pole \( \frac{1}{\Lambda_{\delta}} \) and \( \frac{1}{\Lambda_\sigma} \) are computed as follows:

\[
\Lambda_{\delta} = \mu_0 \frac{\pi L}{4\pi [\ln D_i - \ln (D_i - 2\delta)]}
\]

Where, \( D_i \) is the inner diameter; \( \delta \) is the thickness of airgap.

\[
\Lambda_\sigma = \mu_b \mu_0 \frac{\pi L}{4\pi [\ln D_b - \ln (D_b - 2h_b)]}
\]

Where, \( \mu_b \), \( D_b \) and \( h_b \) are the relative permeability, external diameter and thickness of nonmagnetic hub, respectively.

Stator teeth can be considered to be a partial hollow cylindrical element and its reluctance for each magnetic pole \( \frac{1}{\Lambda_s} \) is given by:
\[ A_x = k_p \mu_s \mu_0 \frac{r L}{4p [\ln D_x - \ln D_y]} \quad (19) \]

Where, \( k_p \) denotes the mean proportion of stator teeth in the perimeter of cross-section; \( \mu_s, D_x \) and \( D_y \) are the relative permeability, external and inner diameter of the stator teeth, respectively. \( \mu_s \) has nonlinear characteristics when IPMSM operates in saturated condition and its value depends on flux density \( B_s \) in stator teeth. In [4], the characteristics of the relative magnetic permeability versus flux density \( \mu_s = 0 \) denoted the initial relative permeability; \( B(\mu_{max}) = 0.75T \) denotes the flux density when \( \mu_r \) reaches the peak, 0.75T; \( c_a = 558790, c_b = 76 \) and \( n = 10 \) are fitting parameters.

The parallel reluctance of \( R_d, R_q \) and \( R_{dq} \) is equivalent to the stator teeth reluctance. The proportion of the three elements should be set properly to describe the saturation effects accurately. The proportion of \( R_{dq} \) represents cross-saturation effects. When cross-saturation effects are neglected, the proportion of \( R_{dq} \) is zero by setting the reluctance to infinite.

### 3.4 EMEC model

This paper presents an EMEC model of PMSM using Modelica based on the equivalent circle model in MSL 3.2.1, as is shown in Figure 14. In order to simulate the fluxes in motor and take into account saturation, two magnetic elements and an electromagnetic element are introduced, and the airgap element is modified.

Figure 14: EMEC model of IPMSM

(1) Magnetic elements: \( R_{\text{bridge}} \) is the reluctance of nonmagnetic hub, \( R_{\text{PM}} \) the PM, \( r_{\text{IronCoreGear}} \) the stator teeth.
(2) \( F_s \) is the flux source of PMs.
(3) EMEF is the element for electro-mechanical transformation.
(4) Airgap element produces the torque based on the magnetic variables, and the calculation formula is:

\[ T_e = 3p^2 (\phi_{sd} F_{sd} - \phi_{dq} F_{dq}) \quad (21) \]

Where, \( \phi_{sd}, \phi_{dq} \) denote the flux under each pole through the airgap in the d- and q-axis frame, respectively; \( F_{sd}, F_{dq} \) denote the MMF of the airgap.

### 4 Verification of EMCM model

The EMEC method was applied to obtain the characteristics of a three-phase IPMSM with star
connection of winding. The main design parameters of this motor are listed in Table 1, and the structure and material parameters concerning magnetic circuits are listed in Table 2.

<table>
<thead>
<tr>
<th>Table 1: Design parameters of PMSM</th>
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<tbody>
<tr>
<td><strong>Quantity</strong></td>
</tr>
<tr>
<td>Maximum torque $T_{\text{max}}$</td>
</tr>
<tr>
<td>Maximum speed $\omega_{\text{max}}$</td>
</tr>
<tr>
<td>Rated speed $\omega_r$</td>
</tr>
<tr>
<td>Maximum DC voltage $U_{\text{dc}}$</td>
</tr>
<tr>
<td>Maximum AC current $I_{\text{pmax}}$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2: Parameters of magnetic circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quantity</strong></td>
</tr>
<tr>
<td>$B_r$</td>
</tr>
<tr>
<td>$\mu_0$</td>
</tr>
<tr>
<td>$\mu_m$</td>
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<tr>
<td>$L$</td>
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<tr>
<td>$e_m$</td>
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<td>$\delta$</td>
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<td>$h_b$</td>
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<tr>
<td>$N_3$</td>
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<tr>
<td>$k_w$</td>
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</table>

A two-dimensional FEM illustrated in Figure 15 is built to validate the accuracy of the EMEC model. Figure 16 shows the field. When the cross saturation is neglected, the inductances calculated by the two models in dq frame are compared in Figure. It is noted that the variation of between the two sets of simulation results is less than 10%. The main reasons for the deviation can be concluded as following:

- The simplification of the reluctance of stator teeth and the waveform of flux densities in airgap.
- The resistance of rotor iron and linkage fluxes are not taken into consideration when modelling.

![Figure 15: FEM of IPMSM](image)

## 5 Applied in a PMSM System Model

The EMEC of IPMSM proposed in this paper is applied to powertrain system model based on Modelica for Hybrid Vehicle Applications. The model of motor system containing control and physical object modules, as is shown in Figure 17.

![Figure 17: Motor system model based on Modelica](image)

The control module play a role in generating the target values of three-phase voltage based on the input signals from powertrain system and specific algorithm for controlling the motor.

![Figure 18: Control module based on Modelica](image)

The physical module of IPMSM consists of the inverter and motor. The inverter model receives the voltage signals and transfers them to electric potentials which feed the motor.
This PMSM system model can be utilized to simulate the motor switching between torque open-loop and revolving speed closed-loop control. Figure 20 shows the revolving speed \( \omega_r \), torque \( T_e \) and stator voltages when motor working under revolving speed closed-loop control.

Figure 20: Revolving speed closed-loop control

6 Conclusion

This paper presents an EMEC model of IPMSM considering saturation, and implementing it using Modelica language. The comparison of inductances from MEC model and FEM indicates the accuracy of the proposed model is enough to system design phase. In the end, the EMEC model is applied to the system model of an electromechanical powertrain to help developing its control algorithms.

References


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